



Course Description

Reasoning About Numbers is designed for teachers who want to deepen and strengthen their mathematical content knowledge and academic background. This course focuses on mathematical reasoning in the context of middle grades number sense: multiplication, multiples, factors and primes, and composite numbers. The math activities are complex and require the teachers to explain their own reasoning. Teachers read the NCTM Reasoning Standard, relate it to their own work and to classroom vignettes, and discuss how the standard relates to assignments, student work and classroom practice. The course uses a collaborative inquiry approach for learning about number sense and reasoning and proof.

Course Objectives

The main goals for this course are for teachers to:

- Use inquiry to move from recognition of patterns to constructing mathematical explanations that have predictive power.
- Construct a mathematical argument to present to others and challenge or confirm someone else's mathematical argument.
- Understand concepts of multiples, factors, primes composites, lowest common multiple, greatest common factor, and square roots, and recognize relationships among them.
- See patterns and relationships in the base ten number system.
- Use factors and the relations they represent to solve problems, and make sense of and explain mathematical patterns.
- Develop and explain one's own algorithms for generating least common multiples; understand and use traditional algorithms.
- Articulate the core ideas in the NCTM's Reasoning and Proof standard and use it to evaluate mathematical activities.

Additional information about assignments

Each week participants review specific content, solve mathematics problems and post their solutions, and, when appropriate, study instances of students' misconceptions about the content. Each week there are particular topics on which participants contribute to online discussions about mathematical content and teaching and learning. Participants are expected to create posts in response to each week's assignment and to read and respond to the posts of at least two colleagues.

Participants keep a Course Journal in which they record mathematical investigations and findings, reflections on their experiences and process, and the potential application of the ideas and materials they have been working with to their own classrooms and curriculum. Teachers make several entries in this journal on a weekly basis.

Because the course is online, in lieu of oral reports participants present their work to group members on a regular basis in a text-based online discussion board. For the most part, work is excerpted from the Course Journal. Developing the capacity to present ideas clearly in writing and participating in an online dialogue that supports learning is an ongoing process that is monitored and supported by online moderators.





C O U R S E O U T L I N E

**WEEK 1 • Reasoning and
Numbers**

During this week, participants are introduced to the NCTM Reasoning and Proof Standard, using multiplication as the context. Using different models of multiplication, they investigate visual and numeric patterns and relationships. Mathematical reasoning is used to develop generalizations about number patterns and consider the conditions under which these generalizations hold true.

Topics & Activities:

1. Course Overview.

Activity: How Did You Learn It? Teachers are asked to reflect upon their own experiences as a learner and Course Journals are started.

2. Mathematical Reasoning.

Activity: Reasoning and Your Standards. Teachers read the mathematics standards for their state or district, noting the standards or references to mathematical reasoning.

3. Number Relationships and Multiplication.

Activity: The Number Array Interactive Tool. This is an introductory activity about multiplication tables which is used to become familiar with the Number Array interactive tool.

4. Reasoning with Multiplication.

Activity: Number Patterns in Multiplication. Using multiplication tables with the Number Array interactive tool, participants make conjectures about number patterns. They also test their conjectures with other numbers.

5. Number Patterns in Multiplication Solutions.

6. Visual Patterns in Multiplication.

Activity: Do You See What I See? Participants show each step of their reasoning and explain why the visual patterns make sense.

7. Generalizations and Conditions.

Activity: Does the Generalization Hold? What happens to a pattern if the chart is changed? Participants check their predictions and explain any consistencies or inconsistencies.

Assignment

Participants create a post drawn from *Activity: Do You See What I See?* They name the table that is being worked on, describe the pattern identified and describe the mathematical explanation for why the pattern appears in that table.

Participants read the patterns and explanations found by their colleagues and provide feedback about whether the explanations make sense and how they are alike and different from their own.



C O U R S E O U T L I N E

WEEK 2 • *Multiples and Factors*

This week centers on patterns in common multiples. Participants gather data on common multiples and use data and observations to create a rule for predicting the least common multiple of any two numbers. They also explore factors and the relationship between factors and divisibility.

Topics & Activities:

1. Common Multiples.

Activity: LCM Patterns and Rules. This activity is about finding least common multiples without making lists. Is it possible to generate a rule that will give the first common multiple of a selected number pair?

2. Uncovered Numbers.

3. Factors and Products.

Activity: Factoring with Arrays. Using arrays and the interactive tool Multiplication and Division with Arrays, participants provide a model for factoring.

4. Numbers of Factors.

Activity: The Locker Problem. The classic problem involves factorization of whole numbers. Participants try solving the problem based on the first 20 lockers and students, and then predict which lockers are open after 1,000 students.

5. Divisibility.

Activity: Your Own Divisibility Rules. This activity uses patterns or reasoning to create rules for divisibility.

Assignment

Participants create a post drawn from *Activity: Your Own Divisibility Rules*. They share one of their own divisibility rules and any explanations they have come up with. Once their rule is posted, participants must read someone else's rule and in response explain why it does or does not make sense.



C O U R S E O U T L I N E

**WEEK 3 • Number Sense
as Tools for
Thinking**

During the week participants continue to use the essential components of conjecture development: gathering data and making observations; describing patterns; making and sharing conjectures; testing conjectures; and revising conjectures. They have a chance to revise their rule for finding least common multiple in light of the ideas about factors and prime numbers.

Topics & Activities:

1. Reasoning, Games, and Strategies.

Activity: The Factor Game, the Product Game. These games are played on paper or on the computer, making participants think about mathematical ideas that are arising and strategies for winning the games.

Activity: Reasoning an Effective Strategy. Participants select either game and reflect on the strategies for winning and the mathematical knowledge drawn on as it was played.

2. More on Factors.

Activity: Uncovered Primes. Participants find the relationship between the largest number whose multiples is needed to cover and the highest prime number left uncovered.

3. Making Connections.

Activity: Refining the LCM Rule. Participants investigate whether the greatest common factor and/or prime factors can be used in finding the least common multiple.

4. Reasoning Your Rules.

Activity: Rules and Reasoning. Participants find posts to compare and contrast and add to the rules and build upon the ideas. They look for new insights or questions after reading the rules posted by other colleagues.

4. Thinking About the Math.

Assignment

Participants create a post drawn from *Activity: Refining LCM rules*.

Some claim the GCF is a tool in finding an LCM. Others say they use primes. The challenge is to investigate these claims and see whether the greatest common factor and/or prime factors can be used in finding the least common multiple and if so, how.

Participants use the Factor Tree applet at the National Library of Virtual Manipulatives website to explore these claims. They record several pairs of numbers and then note the least common multiple and greatest common factor for each. They describe any pattern found in the data that illustrates a relationship between multiples and factors, describe it in their journal entry and post their findings.



C O U R S E O U T L I N E

WEEK 4 • *Number Sequences*

In Week 4

participants investigate number patterns in both square and triangular numbers and make conjectures about why those patterns occur. Based on their observations, they develop a rule for determining the n th square or triangular number in a sequence.

Topics & Activities:

- 1. Reasoning About Number Sequences.**
- 2. Arithmetic Sequences.**
Activity: Explore Arithmetic Sequences. Participants find a rule that relates a number to its term number.
- 3. Geometric Sequences.**
Activity: Consider Your Options. An exploration of exponential vs. linear growth.
Activity: Rules for Geometric Sequences. This is a look at the pattern between the term number and the number in the sequence. Participants try to write a rule.
- 4. Square Numbers.**
Activity: Explore Square Numbers. Participants explore the connection of the numeric pattern and the visual sequence.
Activity: Taking It Further. Participants are offered optional challenge activities.
- 5. Visual Reasoning.**
- 6. Triangular Numbers.**
Activity: Triangles and Squares. How do triangular numbers relate to square numbers?
- 7. Making Generalizations.**
- 8. The n th Triangular Number.**

Assignment

From the questions in the *Activity: Explore Triangle Numbers*, participants post their most interesting finding to the discussion board

They read the posts of their colleagues and look for two that, although they may sound different, seem to reflect a common underlying idea. These observations are posted on the Discussion Board as replies to both original posts.



C O U R S E O U T L I N E

WEEK 5 • *A Look at Students in the Classroom*

During this final week of the course, participants draw on their experiences in reasoning about number and investigate ways in which they can take insights and knowledge gained in the course back to the classroom. Through the use of classroom vignettes they have an opportunity to glimpse what “reasoning about number” might look like with students. Participants consider the teacher’s role in promoting reasoning in the classroom, and examine strategies, questions, and problems designed to promote mathematical reasoning.

Topics & Activities:

- 1. A Window to the Classroom.**
Activity: Video: Reasoning in the Elementary Classroom. 5th grade students use a number array to reason with multiples. Participants watch the video and then reflect on the role of reasoning in the classroom.
Activity: Reasoning in a Middle School Classroom. Participants look back and reflect on what’s been most important to them in this course.
- 2. Research Evidence.**
Activity: Research on Reasoning. Participants read what researchers have to say about aspects and implications of promoting reasoning in mathematical classrooms.
- 3. The Teacher’s Role in Reasoning.**
Activity: Evens and Odds. Participants read the conversation between the student and teacher. They reflect on how to guide the student to use her developing ability to reason mathematically and solve the problem at hand.
- 4. Problems that Promote Reasoning.**
Activity: Solve a Problem. Participants select one of the problems presented and explain a problem solving approach, why the particular strategy was chosen, the mathematical ideas that helped solve the problem, and how students might respond to the problem.
- 5. Summing Up.**
Activity: Ending Reflection. Each participant reflects on what ideas stand out for them, and what questions remain.

Assignment

Drawing from the course journal, participants create a post that is based on the *Ending Reflection* activity.

Participants read the posts of colleagues, look for common themes or new ideas and try to develop two questions that will help extend or deepen the thinking of the group. These are posted as replies to two other posts.

Course Journal Assignment: Submit completed Course Journal that includes all work from weekly activities. (Posts to the discussion boards reflect work on weekly assignments and need not also be included in the Course Journal.)



GRADING & CREDIT

Successful completion of the course for graduate credit is determined by:
Course Moderator/Course Instructor

Grading Overview

1. **(35 points)** Quality and completeness of weekly assignments, as reflected in the posts on discussion board.
2. **(25 points)** Quality of collaborative participation in course discussion.
3. **(40 points)** Quality and completeness of the Course Journal, due at the end of the course, which includes work on all non-optional activities in the Course Documents.

Letter grades are determined by adding the points earned (100 points possible for one graduate credit): 90-100 points = A, 80-89 points = B, 70-79 points = C.

Criteria and Descriptions

Weekly Assignments (35 points = 7 points per weekly assignment)

Posts with participant's responses to the weekly assignment should be well-focused on the questions asked, clear, well-organized, and complete (all assignments and activities are worked through to conclusion).

Because assignments are intended for learning, initial posts will not necessarily show complete understanding of the topic or include full and accurate solutions, but should show increasing understanding over the week.

Collaborative Participation (25 points = 5 points per weekly discussion)

Interactivity (4 points per week) - Challenges or questions other posts; stimulates further discussion; makes multiple, quality replies

Netiquette and Presentation (1 point per week)

Regards most rules of netiquette; includes appropriate grammar, spelling, and punctuation.

Course Journal (35 points for content; 5 points for presentation)

- The Course Journal includes all work on all non-optional activities presented in Course Documents, including one's reasoning or steps in solving mathematical activities.
- Journal entries should include evidence of understanding (or growing understanding) of mathematical concepts in Course Documents.
- Where applicable, final entries on any specific mathematics activities are reasonable and present accurate solutions. (N.B.: Initial entries may include errors in computation or strategy, but subsequent entries should show evidence of mathematical growth and understanding.)
- Where applicable, shows reflection or analysis of teaching practice, application of ideas to student learning, and understanding of standards.

Journal Presentation: Journal should be well organized and legible, with entries clearly labeled with dates and assignment titles. Tone and style can be informal, but correct grammar, spelling, and punctuation should be used.



Reading List or Bibliography, and Supplementary Materials

Books

Chapin, S. & Johnson, A. (2000). *Math matters: understanding the math you teach, Grades K-6*. Sausalito, CA: Math Solutions Publications.

COMAP. (2002). *The ARC Center tri-state student achievement study*. Lexington, MA: Consortium for Mathematics and Its Applications.

Driscoll, M. (1999). *Fostering algebraic thinking: guide for teachers grades 6-10*. Portsmouth, NH: Heinemann.

Krulik, S. and Rudnick, J. A. (1999). Innovative tasks to improve critical- and creative-thinking skills. In L.V. Stiff & F.R. Curcio (Eds.), *Developing mathematical reasoning in grades K-12, 1999 yearbook* (pp. 138-145). Reston, VA: National Council of Teachers of Mathematics.

Malloy, C. (1999). Developing mathematical reasoning in the middle grades. In L.V. Stiff & F.R. Curcio (Eds.), *Developing mathematical reasoning in grades K-12, 1999 yearbook*. Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Richards, S. P. (1982). *A number for your thoughts*. New Providence, NJ: SP Richards.

Russell, S. J. (1999). Mathematical reasoning in the elementary grades. In L.V. Stiff & F.R. Curcio (Eds.), *Developing mathematical reasoning in grades K-12, 1999 yearbook* (pp. 1-12). Reston, VA: National Council of Teachers of Mathematics.

Sconiers, S. (1998). Patterns on the 100s Board [Student vignette video]. In S. Sconiers, *Bridges to Classroom Mathematics: a Staff-development Curriculum for Elementary School Teachers*. Lexington, MA: COMAP.

Steen, L. A. (1999). Twenty questions about mathematical reasoning. In L.V. Stiff & F. R. Curcio (Eds.), *Developing mathematical reasoning in grades K-12, 1999 yearbook* (pp. 270-282). Reston, VA: National Council of Teachers of Mathematics.

Web Resources

The ARC Center (COMAP)
<http://www.comap.com/elementary/projects/arc/>

The Factor Game
http://illuminations.nctm.org/tools/tool_detail.aspx?id=12

Factor Tree Applet
http://matti.usu.edu/nlvm/nav/frames_asid_202_g_3_t_1.html

National Council of Teachers of Mathematics
<http://www.nctm.org>

NCTM Principles and Standards for School Mathematics
<http://standards.nctm.org/document/index.htm>

National Library of Virtual Manipulatives
<http://matti.usu.edu/nlvm/nav/index.html>

Product Game
http://illuminations.nctm.org/tools/tool_detail.aspx?id=29

Sieve of Eratosthenes
<http://www.faust.fr.bw.schule.de/mhb/eratosiv.htm>

About the Instructor

Education

- Ph.D. (Education/Psychology), Massachusetts Institute of Technology, Cambridge, MA 1994
- M.Ed., Lesley College, Cambridge, MA 1980
- Massachusetts Elementary Teaching Certificate, Shady Hill School Apprentice Teaching Program, Cambridge, MA 1971
- BA (Psychology), Vassar College, Poughkeepsie, NY 1970

Professional Positions

- Associate Professor School of Education Lesley University, Cambridge, MA, 1998-present
- Senior Scientist, GTE/BBN Technologies, Cambridge, MA, 1990-1998
- Assistant Professor Graduate Education, Lesley College, Cambridge, MA, 1981-1990

Professional Responsibilities and Projects

Associate Professor School of Education Lesley College
Focus of current work: research, development and teaching to support professional development of teachers using online, computer, communications and video technologies.

Current Funded Projects:

- Director of Mathematical Professional Development Mathematica Project (Department of Education Star Schools Project) (2001-present)
- National Advisory Board for Seeing Math Telecommunications Project - online video case studies for professional development. (2001-present)

Previous Projects:

- Web Consultant and researcher for the ARC Center. NSF funded dissemination center for standards based elementary curriculum. (1999-2001)
- Middle School Mathematics Program , EDC/Everyday Learning (Author/Consultant,1998- 2000)
- On-line Collaboration for Reform Project for the Regional Education LAB at Brown. (Consultant, 1995-1999)
- IMPACT Learning Network for Northeastern's IMPACT Curriculum Dissemination and Implementation Center (Director, subcontract to NSF project, 1998-1999)
- The Mathematical Inquiry Videotapes: Tools for Professional Growth. (PI, NSF, 1994-1998)
- Vanguard For Learning Project, a systemic reform research initiative being carried out in Department of Defense schools. (Director of Operations, NSF, 1996-1998)
- Empowering Teachers Mathematical Inquiry Through Technology: developed inquiry oriented, technology based algebra and geometry courses for middle school mathematics teachers. Included distance learning and video development. (PI, NSF, 1991-1996)
- Co-NECT Schools (Chair of Staff Development group, New American Schools Development Corporation, 1992-1994).

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Curriculum Vitae June, 2002

- Resource Center Coordinator, Lincoln Public Schools, Lincoln, MA, 1974-1979
- Writer and Consultant on Computers in Education

Assistant Professor in Graduate Education Lesley College 1982-1990
Work at Lesley included: teaching in Graduate, Outreach, and Undergraduate programs, directing Educational Computing Lab, working on NSF funded projects.

Other Funded Projects:

- Intelligent Tools for Mathematical Inquiry. (Director of Implementation, NSF, 1988-1991) Responsibilities: managing curriculum development; pilot training; evaluation; implementation.
- Logo Algebra Project. (Senior Researcher, NSF, 1985-1987). Responsibilities included: developing software and curriculum, teaching 6th grade algebra course. This work was the basis for doctoral dissertation on student understanding of variables.

Resource Center Coordinator for Lincoln Public Schools
Responsibilities included: staff development, program development, and curriculum development as well as daily work with students. As a Math Curriculum Committee member developed K-8 math curriculum including the introduction of microcomputers.

Other Professional Activities

- Program Chair for National Educational Computing Conference, Boston, 1994
- Developed and hosted MCET distance learning TV show: Middle School Inquiry: Math and Technology 1992-94
- Faculty Member SummerMath for Teachers 1992
- Administrative Director and Faculty Member, "Logo Institute" 1988-1989
- Developer and Director of Lesley College Computer Center, 1980-1985
- Reviewed manuscripts for Journal of Learning Sciences, West, Little Brown, Prentice Hall, Houghton Mifflin, Wadsworth, Allwyn and Bacon, and MIT Press.



About the Instructor

Selected Publications

- Arshavsky, N., Carter, R., Foster S., Janssen, S., Lukas, J., Manes, M., Orrell, C., Ruopp, F., Watt, D. Impact Mathematics - Algebra and More for the Middle Grades. Chicago Illinois, Everyday Learning. 2000
- Carter, R. Harik, F. Mathematical Inquiry Through Video (ten video case studies of middle school math teachers trying to change their practice) (currently in negotiation with publisher)
- Carter, R. et al. Electronic Collaboration: A Practical Guide for Educators , Providence RI, The LAB at Brown University, 1999
- Carter, R., Richards, J. "Dilemmas of Constructivist Mathematics Teaching: Instances from Classroom Practice" in Mathematics Teacher Education: Critical International Perspectives. Jaworski, B., Wood, T. & Dawson, A.J. London: Falmer Press 1999
- Roberts, N., Carter, R.C., Davis, R., and Feurzeig, W. "Power Tools for Algebra Problem Solving," The Journal of Mathematical Behavior, Vol. 8, pgs. 251-265, 1989.
- Roberts, N., Carter, R.C., Friel, S., and Miller, A. Integrating Computers into the Elementary School Curriculum, Englewood Cliffs, N.J.: Prentice Hall, 1987.
- Carter, R.C. "Teacher Talk as a Tool for Effective Research," in Stubbs, M.T. Amabile (Eds.) Psychological Research in

Selected Presentations and Talks

- Invited speaker. American Association for Colleges of Teacher Education Forum on Technology: Technology in Education: The Promise versus the Reality. 2001
- National Council of Teachers of Mathematics annual conference presentations 1998-2002
- National Educational Computing Conference presentations 1987-1994, 2001-2002
- International Conference of Psychology of Math Education 1992. Chaired Study Group on "Dilemmas of Inquiry Teaching"
- Lesley College Annual Computer Conference 1992 Keynote speaker: "Schooldays 1991 revisited" (Awarded Renaissance Person 1992 Lesley College)
- Lecture tour in Sweden, lecturing on Logo for Swedish National Education Board, 1984

